Risk Measurement for Project Finance Guarantees

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When embarking on a guarantees program, the guarantor confronts several issues:

- It must decide whether to give a guarantee to a particular project.
- It must decide the terms, structure, and pricing of the transaction.
- It must decide how to budget and reserve for possible losses.

There is a maxim in risk management that says, "You can't manage what you can't measure." To properly manage the guarantees program, the guarantor must have good risk measurement tools. The tools should give guidance in the following areas:

- How much could be lost from the guarantee?
- What circumstances would cause that loss?
- Are those losses likely to come at a time when the guarantor is facing large payments for other liabilities?
- If the guarantor sets up a reserve to take care of the losses, how much should be in that reserve?
- What should be the source and timing for payments into the reserve?
- Should the guarantor expect the reserve to grow or be depleted over time?
- How much should the guarantor charge the project company that benefits from the guarantee?
- How does the cost of the guarantee compare with the cost of other investments?
- Can the guarantee be restructured to shift some of the risks away from the guarantor?

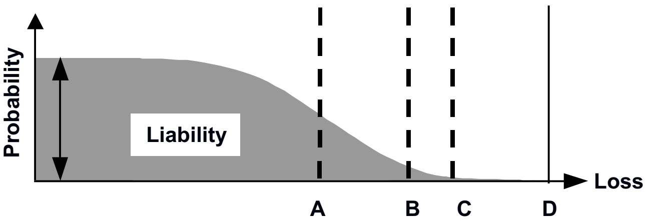
To answer these questions, the guarantor must know the risks and costs associated with the guarantee. Ideally, the risks and costs should be stated in a form that allows the fiscal implications of the guarantee to be compared on an apples-to-apples basis with other commitments such as direct loans and subsidies. The framework discussed in this paper can be used to answer all of the above questions. It is sufficiently fundamental that it can also incorporate direct liabilities, guarantees, and other contingent liabilities.

METHODOLOGICAL FRAMEWORK

Use of the Cumulative Probability Function

With a guarantee program, as with many other liabilities, the guarantor cannot be certain of the amount that it will pay each year. This uncertainty can be represented by a cumulative probability function (CPF). The cumulative probability function for the loss from a portfolio of risks can be used by the guarantor to determine how much it should budget as a reserve. Exhibit 1 shows an example of a CPF.

EXHIBIT 1
Illustration of a Cumulative Probability Function



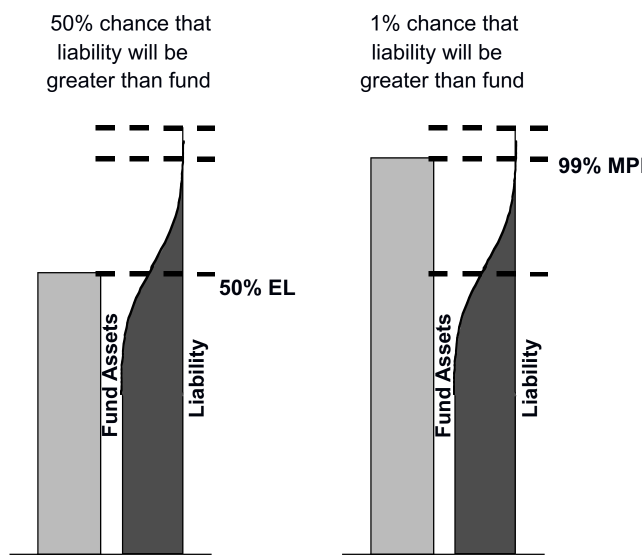
The height of the CPF on the y-axis tells us the probability that the loss could be greater than the value shown along the x-axis. For example, this graph shows that there is approximately a 50% chance that the loss will be more than A. There is a 5% chance that it could be greater than B, a 1% chance that the loss could be greater than C, and 0% chance that it is greater than D. We label these points as follows:

A Expected Loss (EL)
B 95% Maximum Probable Loss (95% MPL)
C 99% Maximum Probable Loss (99% MPL)
D Maximum Loss (ML)

For any probability distribution, the average outcome is the expected loss, but typically the actual result will be greater or less than EL. To ensure that the reserve is not exhausted, its size must equal the maximum probable loss of the risk. Depending on risk aversion or the guarantor's credit rating, the reserve fund would be set equal to 95% of MPL, 99% of MPL, 99.9% of MPL, etc. For example, if the guarantor set the assets in the reserve to be equal to the expected loss, there would be approximately[1] a 50% chance that all the required payments could be made from the reserve. If the guarantor wanted there to be a 99% chance that the reserve would be sufficient, the reserve must be equal to the 99% MPL of the risk. This is illustrated in Exhibit 2.

Normally the guarantor would want to ensure there was a high probability that the fund would not be exhausted. For that to be the case, the fund must cover the maximum probable loss, not just the average expected loss. The choice of confidence level (e.g., 95% or 99%, etc.) is a matter for serious policy debate and depends on how the guarantor wants to view the reserve[2] as discussed later. For the rest of this paper a 99% MPL will be used.

EXHIBIT 2
Illustration of the Relationship Between the Level of the Reserve Fund and the Probability of Its Being Exhausted



Calculation of the Expected Loss and Maximum Probable Loss

The problem now is to determine the form of the CPF for a given set of liabilities. The total payment or loss from the portfolio is the sum of the losses for each of the individual liabilities. Similarly, the average expected loss from the portfolio is simply the sum of the expected loss for each of the individual liabilities:

$$EL_{Portfolio} = \sum_{i=1}^{N} EL_i$$

So far, the problem seems straightforward. However, complexity arises because the maximum probable loss for the portfolio does not necessarily equal the sum of the MPLs for each liability.[3]

$$MPL_{Portfolio} \neq \sum_{i=1}^{N} MPL_i$$

This nonadditive nature of the MPLs occurs because of correlation and diversification effects. The correlation effect is illustrated in Exhibit 3 where there are two portfolios, each with the same EL, but with different MPLs. The two telecoms projects together in a portfolio add with highly correlated losses to cause a high MPL. The toll road and telecoms projects combined in a portfolio diversify each other to reduce the MPL.

From these considerations of portfolio correlation effects, we can see that the "cost" of having highly correlated projects is captured in the MPL of the portfolio. If the projects are highly correlated, the MPL will be higher, the reserve will be higher, and the "cost" of that reserve will be higher.

Commercial Pricing for a Portfolio of Guarantees

If a bank only charged enough to cover the expected loss, it would, on average, break even. However, there would be no compensation for the shareholders who risked investing money to create the initial capital as the cushion to absorb the maximum probable loss in case of a bad year. Banks therefore charge their customers enough to cover the EL plus a percentage return for the shareholders who put their money at risk to provide the cushion for losses beyond EL. For the portfolio as a whole, a bank would price as follows:[4]

$$Price_P = EL_P + \left(MPL_P - EL_P\right) \times H + OC_P$$

Here the subscript P denotes the portfolio and H is the hurdle rate return demanded by the shareholders for their risky equity investment, typically around 20%-30% per year. OC represents the overhead and personnel costs.

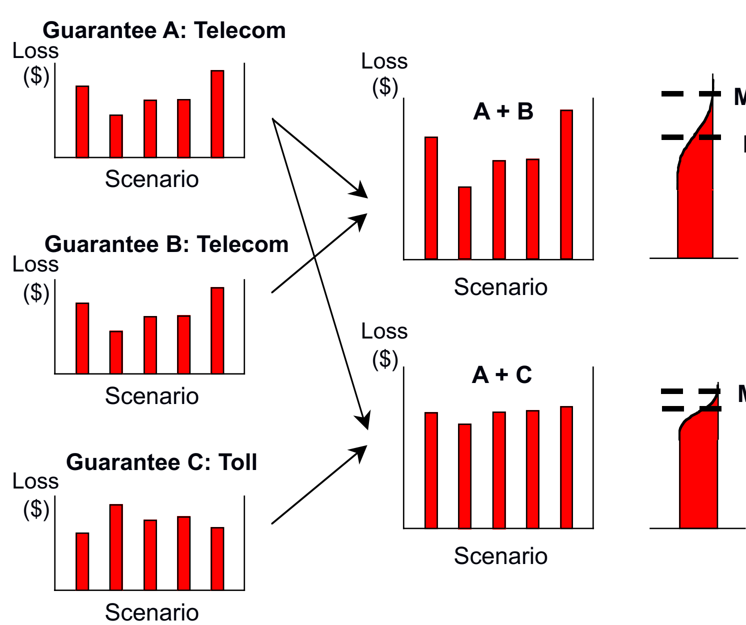
This equation sets the price for the portfolio as a whole. The individual projects are charged according to their contribution to the portfolio's EL, MPL, and OC. The EL for the portfolio is simply the sum of EL for the individual guarantees, and therefore the EL contribution is simply the EL for the guarantee. The OC can be allocated using activity-based costing and adds up to the total operating cost for the portfolio. However, the allocation of MPL is more complex.

Allocation of Maximum Probable Loss Contribution

Each individual project should be charged according to its contribution to the portfolio's MPL. The project's MPL contribution is usually less than the project's stand-alone MPL because of diversification effects:

$$MPLC_i \leq MPL_i$$

EXHIBIT 3
Illustration of Correlation Effects



As explained later, a simulation method is used to evaluate the project under various random scenarios. With the simulation method, we can allocate the maximum probable loss using only the scenarios in which the reserve is exhausted. On each occasion when the reserve is exhausted, we record the loss contribution from the individual projects. We then define the maximum probable loss contribution (MPLC) for project i to be the MPL for the portfolio multiplied by the percentage contribution of the individual project:

$$MPLC_i = MPL_p \times \ \%Contribution\,of\ Project_i$$

This has the property that the MPLCs sum up to the MPL of the portfolio:

$$MPL_P = \sum_{i=1}^{N} MPLC_i$$

Pricing for an Individual Guarantee

It is necessary to distinguish between the amount held in the reserve and the amount that is charged to each project. The size of the reserve is MPL. The price for the portfolio as a whole has a component to cover the expected loss and a component to give a return on the capital being held against the possibility of the maximum probable loss for the portfolio. The price was given earlier to be:

$$Price_P = EL_P + \left(MPL_P - EL_P\right) \times H + OC_P$$

The price charged to each project is based on the allocation of EL, MPL, and OC:

$$Price_i = EL_i + \left(MPLC_i - EL_i\right) \times H + OC_i$$

This has the property that the prices for each project sum up to the total required price for the portfolio:

$$Price_P = \sum_{i=1}^{N} Price_i$$

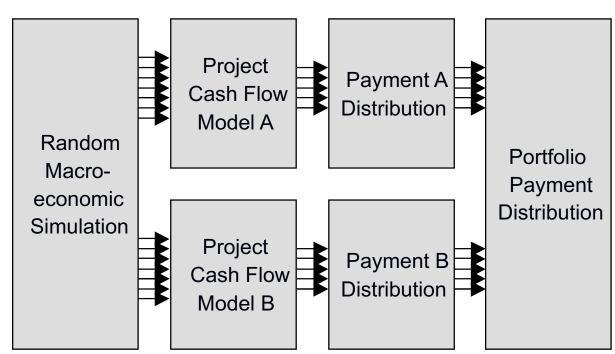
These are the prices that an institution should charge to give the guarantee at a commercial rate. The guarantor may decide to charge a lower amount, thus effectively giving a subsidy. This is discussed in the later section on policy implications.

SIMULATION OF LOSS STATISTICS

Project contracts produce complex, uncertain cash flows. It is extremely difficult analytically to calculate the distribution of these cash flows. A more tractable solution is to use Monte Carlo evaluation, which is described in Marrison [1995]. Monte Carlo uses computer-generated random scenarios. For each scenario, the cash flow consequences are calculated, the results are recorded, and the process repeated. For the examples in this paper, the evaluations are repeated 1,000 times. The simulation works as follows: a scenario is created, every project is evaluated under that scenario, the results are added together to get the portfolio loss, and then the process is repeated for the next scenario. The results allow us to calculate the correlation among any projects by seeing if for any given scenario, all the projects have large payments at the same time, or if they all pay under different scenarios.

The model can be thought to have four components: random scenario generation, cash flow calculation, a statistical reporting section that brings together the results for each project, and a section that brings together the results for the portfolio. The result is that we get the cumulative probability function for each of the projects and for the portfolio as a whole. This process is sketched in Exhibit 4.

EXHIBIT 4
Illustration of Simulation Process



POLICY APPLICATION OF RISK MEASUREMENT

The primary reason for measuring the risk of guarantees is to ensure that sufficient reserves are set aside to avoid crises in the future. For this we require that the NPV of the fund be greater than or equal to the NPV of the maximum probable losses. Furthermore, for prudent liquidity management, we should require that at the beginning of each budget period, the amount in the fund is equal to the maximum probable payments in that year.

Payments to create the fund come from four potential sources: equity from the guarantor, subsidies from the government or agency, fees charged to the projects benefiting from the guarantees, and equity from private investors.

The grant of equity to the fund from the guarantor could be a one-time capitalization. If the fund was run on a commercial basis, the guarantor should expect that the equity should grow each year at the market rate for equity. If the fund was eventually closed in year T, the guarantor would expect, on average, to receive back an amount equal to:

$$E_T = E_o(1+H)^T$$

Here E_0 is the initial equity investment to cover MPL and H is the hurdle rate equal to the average market return on equity of the same riskiness.

If the fund was run on a commercial basis, the fees charged to the projects would cover the expected loss plus a return on the equity being held in case of the maximum probable loss. For a one-year horizon, the commercial price for an individual guarantee is as follows:

$$Commercial\ Price_i = EL_i + \left(MPL_i - EL_i\right)?H + OC_i$$

Here H is the hurdle rate set for the return on equity. This price may be received as an initial cash payment, as a monthly fee, or as a spread on an associated loan.

If the guarantor charges less than the theoretical commercial price, it is giving an implicit subsidy to the project:

$$Subsidy = Commercial\ Price - Actual\ Fee$$

To see that this subsidy is real, consider that, given an amount of equity, the guarantor can choose to invest it in the guarantee fund or find an investment with the same level of risk in the capital markets. If invested in the market, the equity will be expected to make a full market return. If it is invested for a subsidized project, the equity will make less than a market return and this is a genuine loss to the guarantor's shareholders or taxpayers: for the same risk they are getting a lower payback.

Once this subsidy is recognized, it has two uses in policy setting. One is that the guarantor can compare this subsidy with other payments that it makes such as to build schools. It then can weigh the relative costs and benefits of the alternative investments. The other use is that the guarantor may choose to replace the guarantee with a direct subsidy, which has the benefits of transparency and certainty.

The form of financing will be chosen to minimize the moral hazards and maintain the project financing principle that each risk should be allocated to the party most capable of managing that risk. For example, the options could include the following:

- Reprice the full guarantee.
- Make a partial-risk guarantee.
- Give a direct subsidy.
- Move the risk of the guarantee onto the private market.

If the subsidy is zero, then there is no net cost to the guarantor. This opens the discussion as to why the guarantor is guaranteeing a project that can pay full commercial rates. The guarantor would be justified in making such a guarantee either if there is a market failure to provide the guarantee or if the guarantor has an influence over the risk being guaranteed. In this case the guarantee acts as a promise by the guarantor not to upset the project. For example, this would be the case if a government guaranteed that the project would not suffer from changes in environmental law.

The guarantee will have a fundamentally lower cost to a government than to banks if the guarantee has a lower correlation to the government's portfolio than to the banks' portfolios. For example, if banks are already highly exposed to telecoms but payments to telecoms are positively correlated with receipts from taxpayers, it would make sense for the government to take on the guarantee, because when payments are due, tax incomes will be high. However, if payments to telecoms were negatively correlated with tax income, it would be more risky for the government because payments would be due when tax receipts were low.

We have been discussing the guarantee risks as if they were bundled into a portfolio and supported by a stand- alone reserve fund. In fact, in many cases such reserve funds are part of a government's general portfolio and if the fund is exhausted, the excess liability falls on the government. In this case, the portfolio considered should properly be the government's whole budget. The confidence level chosen for MPL would correspond to the government's own debt rating. For example, if the government was rated "A," the required confidence level would be 99.9%.

One possible alternative would be to set up the reserve fund as a legally separate entity run on a commercial basis. This would have three advantages:

- If the fund was exhausted, calls on the guarantor could be limited.
- Private investors could buy equity in the fund, forcing it to be commercially run and reducing the amount of equity initially invested by the guarantor.
- From a legally separate entity, it may be possible to securitize seasoned liabilities into packages that can be passed on to private investors and off the guarantor's books. The World Bank Group's IFC has done a similar packaging of its project finance loans.

If the guarantor still wished to subsidize projects, it could do so with specific subsidy payments made directly to the fund or the individual projects. Such a legally separate fund would work best if the guarantor were truly able to walk away in the event the fund was exhausted. Rather than have the government create this fund, perhaps private banks should create it with limited government subsidies and backing.

The creation of such a fund would require clear understanding and transparency of the risks involved. Such an understanding must be based on risk measurement.

ILLUSTRATIVE EXAMPLES

To illustrate how the risk measurement methodology works, this section describes the results for two simplified examples. The example takes a combination of risks from three sources: a guarantee to a toll road, a guarantee to a bank, and a set of general risks from a large portfolio. The large portfolio represents a set of risks from other undertakings such as many other guarantees, loans, or expected tax receipts. The reason for including the large general portfolio is to illustrate the correlation and diversification effects. The results show the risk statistics on a stand-alone basis *and* as part of a portfolio.

The models used here are very simplified. The main purpose of these examples is to show both how cash flow models can be randomized and how the results can be brought together to create portfolio-level statistics. The individual cash flow models should not be seen as being complete representations of the cash flow structure for banks and toll roads. For this approach to be used in practice, the guarantor would use fully detailed cash flow models.

These illustrative models only consider the continuous cash flow changes caused by fluctuations in market and economic variables. They do not consider disruptive events such as project failure, disaster, or default of the project counterparties. Such disruptive events can be accommodated in the framework and are important to include if the guarantee is contingent upon the event.

Specification for the Macroeconomic Model

The simple examples for this paper are such that the guarantee payments depend on project-specific risks such as construction and on three macroeconomic variables: GDP, inflation, and interest rates. These three factors (and their derived statistics such as CPI) are modeled as random variables.

For this example, inflation and the real interest rate are both modeled with the Cox-Ingersol-Ross process. GDP growth is modeled as a normal random walk with decay towards a mean. Exhibit 5 shows the evolution of GDP growth, inflation, and nominal interest rates for

EXHIBIT 5
Sample Macroeconomic Evolution for One Scenario

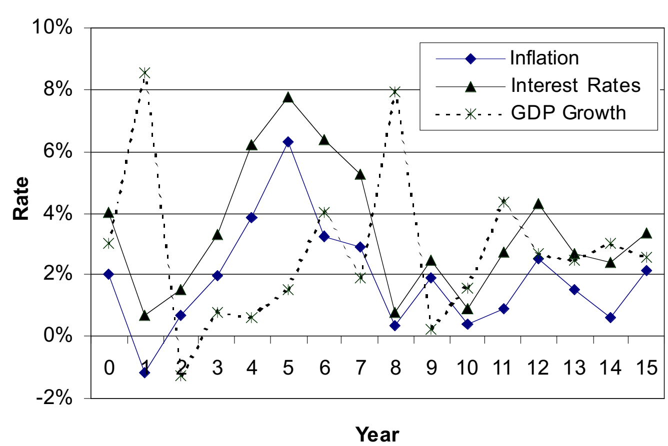


EXHIBIT 6
The Effect of GDP on Guarantee Payments

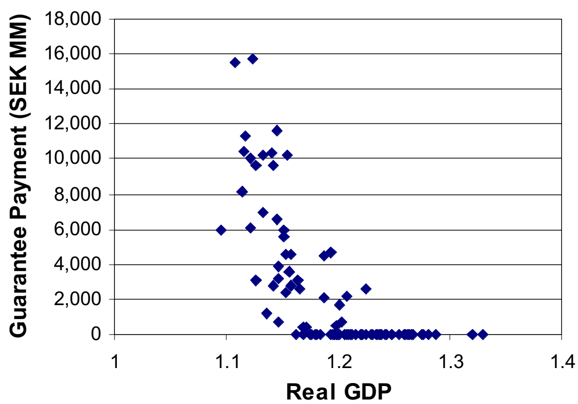
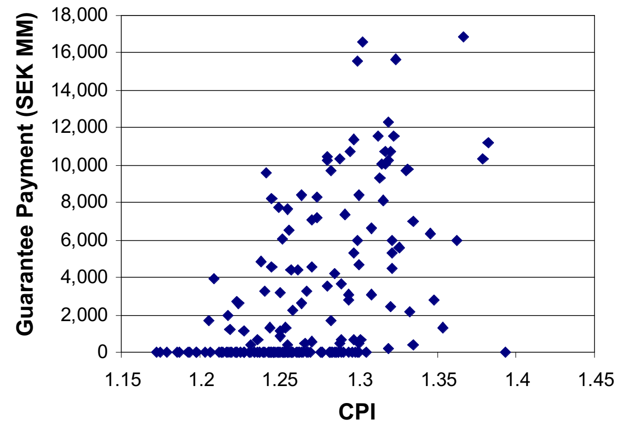


EXHIBIT 7
The Effect of CPI on Guarantee Payments



one of the 1,000 scenarios. It shows the positive correlation between inflation and interest rates and the negative correlation between inflation and GDP growth. It also illustrates the mean-reverting nature of the random walks.

Specification for the Bank Example

A newly formed company (the bad bank) is established with SEK 3 billion of equity capital from private sources and with state guarantees for borrowing SEK 22 billion. Using these guarantees, the bank raises SEK 22 billion on the market and then buys impaired loan assets costing SEK 25 billion from the "good bank." The face value of these assets is SEK 30 billion (this assumes that the recovery rate will be around 80%). At the end of six years, the bad bank will be closed down and the market borrowings of SEK 22 billion will be paid back. If the assets are worth less than SEK 22 billion, the government will make up the difference under the guarantee.

The assets are worth between approximately SEK 3 billion and SEK 30 billion in different scenarios. The SEK 30 billion figure is linked to a continuing strong boom through six years (low interest rate, low inflation, and high GDP-growth rate) and the SEK 3 billion to a continuing recession (high interest rates, etc.). The asset model produces default rates based on changes in GDP and CPI.

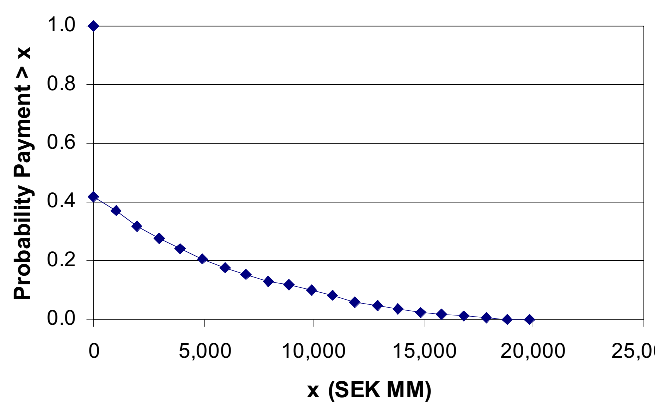
Results for the Bank Example

Exhibits 6 and 7 show the effect of GDP[5] and CPI in year 6 on the guarantee payments. As GDP increases, the required payments decrease; as inflation increases, the payments increase.

EXHIBIT 8
Statistics for the Bank Guarantee

Bank	SEK MM
Average Loss	2,480
Standard Deviation of Loss	4,189
99% Maximum Probable Loss	17,391

EXHIBIT 9
Cumulative Probability Function for Guarantee Payments



The statistics for the payments made under the guarantee are shown in Exhibit 8.

The cumulative probability function for the guarantee payments to the bank is shown in Exhibit 9. There is a 58% probability that the payment will be zero (hence the spike in probability at zero).

Specification for the Toll Road Example

A special company is formed with a low equity capital of SEK 1 billion and provided with state guarantees for its debt financing. The construction cost is uncertain due to poorly known ground conditions—it may vary from approximately SEK 5 billion to SEK 20 billion. The construction cost is therefore an independent random variable, uncorrelated with the economy. The construction cost is given a lognormal distribution. The lognormal distribution has the effect of making cost overruns more likely than cost underruns. The traffic volume (and therefore revenues) vary with GDP and growth in the local population.

The SEK 1 billion equity is provided from the private sector. The construction company will raise debt to pay for any construction costs beyond SEK 1 billion. The debt will be repaid simply in equal installments over 15 years after construction is completed.

The debt will be paid out of traffic toll revenues minus operating costs. If there is insufficient income in any year, the government will pay the banks the difference between the debt payment and the net income. Any excess income in the year will go to the equity investors. (This is a simplification; normal projects would require some earnings to be retained as a cushion for debt holders in the next year.)

Because, in this example, the payments are fixed, the project should become more profitable as the population grows and tolls rise with inflation. The most difficult years are those immediately after construction.

Results for the Toll Road Model

Exhibit 10 shows the guarantee payments as a function of the construction cost. The graph shows that for the project sponsors, the guarantee tends to act as a call option on the cost of construction. When the construction cost is less than approximately SEK 12 billion, the guarantee payments will be very low and only occur if there is unusually low traffic. When construction costs are more than SEK 12 billion, payments tend to increase linearly with construction cost. The slope is less than 45 degrees. This is due to some of the extra construction costs that are absorbed by delayed company profit in the later years.

GDP and CPI have negligible effects on payments. The important conclusion for this illustration is that losses to the guarantor on the toll road guarantee are not strongly correlated with macroeconomic conditions. Toll road losses are more strongly driven by the construction cost, which is uncorrelated with other risks. As we will see in the discussion of the larger portfolio, this has important consequences for the maximum probable loss contribution of the toll road. The summary statistics for the toll road guarantee are shown in Exhibit 11.

Exhibit 12 shows the cumulative probability distribution for the NPV of the guarantee payments. There is a 13% chance that the payment will be zero.

EXHIBIT 10
Relationship Between Construction Cost and Guarantee Payments

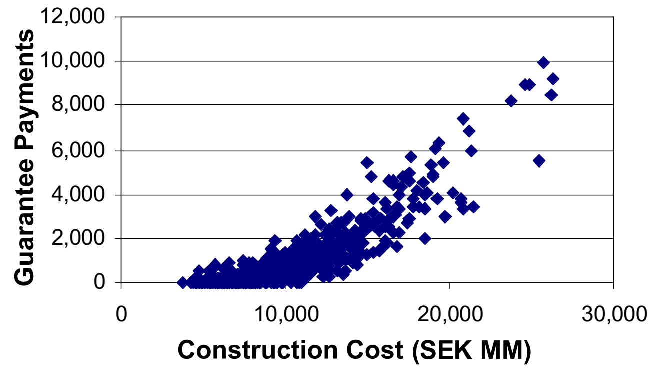


EXHIBIT 11
Statistics for the Toll Road Guarantee

Toll Road	SEK MM
Average Loss	1,024
Standard Deviation of Loss	1,349
99% Maximum Probable Loss	7,226

EXHIBIT 12
Cumulative Probability Function for Guarantee Payment

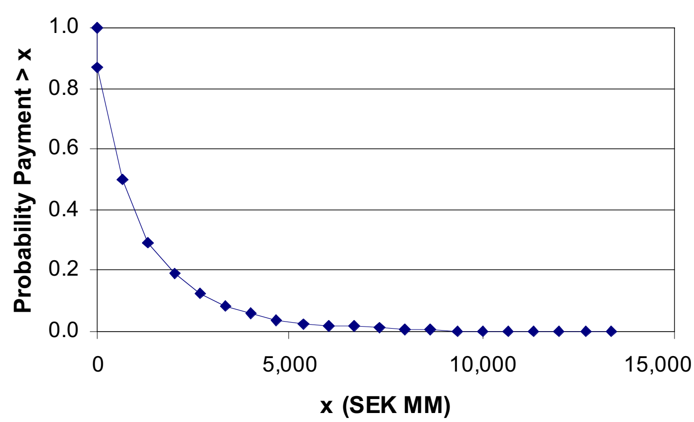


EXHIBIT 13
Profile of Payments over Time

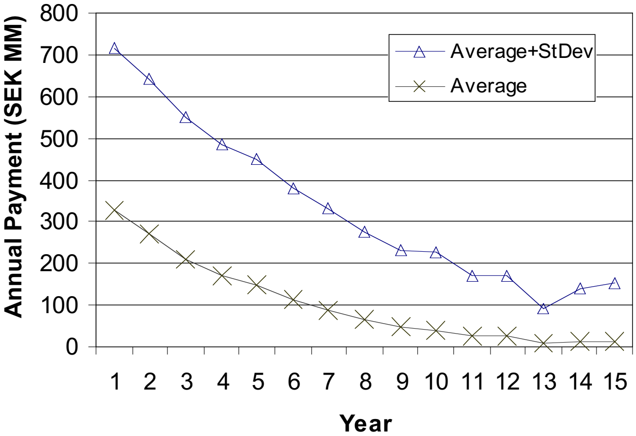


Exhibit 13 shows the profile of guarantee payments over time. The graph shows the average payments and the average plus one standard deviation. The payments drop over time as a result of inflation and as the population rises so that the fixed debt payments can be paid from toll revenue. This graph is useful in telling the guarantor when the losses are likely to occur, and how much should be in the reserve each year.

Specification for the Large Portfolio

The large portfolio represents a centralized set of liabilities held by the guarantor. These may be guarantees, pensions, or any other set of payments. In a full implementation, the structure of the portfolio model would be based on the liabilities in the portfolio. For this illustration a form was chosen such that payments increase at times of country stress and decrease at other times. When GDP and GDP growth are high, payments from the portfolio are low. When inflation is high, payments are high.

Results for the Large Portfolio

The statistics for the NPV of payments from the fund are shown in Exhibit 14. Exhibit 15 shows the cumulative probability function.

EXHIBIT 14
Summary Statistics for the Large Portfolio

Large Portfolio	SEK MM
Average Loss	283,571
Standard Deviation of Loss	50,301
99% Maximum Probable Loss	400,004

EXHIBIT 15
Cumulative Probability Function for the Large Portfolio

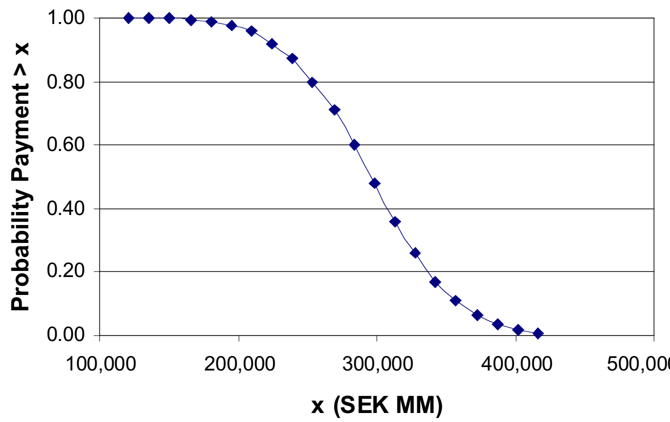


EXHIBIT 16
Summary Statistics for the Combined Portfolio

Combined Portfolio	SEK MM
Average Loss	287,075
Standard Deviation of Loss	52,037
99% Maximum Probable Loss	409,169

Combined Portfolio Results

The combined portfolio is the sum of the losses from the bank guarantee, the toll road guarantee, and the large portfolio. The statistics for the combined portfolio are dominated by the large portfolio. Therefore, the probability distribution and statistics in Exhibit 16 are almost the same as the large portfolio.

Exhibit 17 shows the statistics for each of the three components of the combined portfolio plus the combined

EXHIBIT 17
Statistics for All Components

	Bank	Toll Road	Large Portfolio	Combined
Average Loss	2,480	1,024	283,571	287,075
Standard Deviation of Loss	4,189	1,349	50,301	52,037
99% Maximum Probable Loss	17,391	7,226	400,004	409,169
Maximum Probable Loss Contribution	9,693	1,768	397,708	409,169
Price	3,923	1,173	306,399	311,494

portfolio itself. The portfolio correlation effects are also reflected here in the MPLC and price. Notice that the price is more than the EL, but much less than the project's stand-alone MPL.

CONCLUSIONS

In the introduction, we said that guarantors should have tools to enable them to answer key questions regarding the guarantees. The illustrative models have shown how to answer these questions:

How much could be lost from the guarantee?
The loss is described by the EL and MPL.

What circumstances would cause that loss?
The drivers of loss are shown through the intermediate results. For example, Exhibits 6 and 7 show that the losses for the bank guarantee are driven by GDP and inflation. Exhibit 10 shows that losses for the toll road guarantee are driven primarily by construction costs.

Are those losses likely to come at a time when the guarantor is facing large payments for other liabilities?
This question is answered by the correlation and maximum loss contribution. For example, the correlation between the losses to the bank and the losses on the large portfolio show that the bank's losses are likely to come at times of general stress, whereas the toll road losses are not.

If the guarantor sets up a reserve to take care of the losses, how large should the reserve be?
If the reserve is legally separate from the guarantor, it should equal the MPL for the portfolio of risks. If the guarantor is bound to accept any additional losses from the portfolio, the reserve should equal the MPLC of the portfolio relative to the guarantor's budget.

What should be the source and timing for payments into the reserve?
The guarantor should expect to give the initial capital (equal to the portfolio MPL). The project companies should pay the commercial price into the fund. If the companies are allowed to pay less than this commercial rate, the guarantor should pay into the fund an amount equal to the effective subsidy. The timing of payments must be such that they cover the annual MPL given by the cash flow profile.

Should the guarantor expect the reserve to grow or be depleted over time?
If the pricing and subsidies are made at the theoretical commercial rate, the guarantor should expect the equity in the fund to grow at the hurdle rate. If the guarantor wishes to reduce the guarantee program in the future, it can take money from the fund in the form of dividends.

How much should the guarantor charge the project company that benefits from the guarantee?
The project companies should pay fees equal to the EL plus the MPLC for the project times a hurdle rate (e.g., 20%) plus the operating costs minus any allowed subsidy:

$$Fee = EL + \left(MPLC - EL\right) \times H + OC - Subsidy$$

How does the cost of the guarantee compare with the cost of other investments?
This is shown by the calculation of the effective subsidy.

Can the guarantee be restructured to shift some of the risks away from the guarantor?
Having run the models and found the source of risks, the guarantor can choose to restructure the project to move

some of the risks to the private sector. The guarantor should be most interested in shedding those risks that are found to be highly correlated with losses in the rest of the portfolio. After restructuring, the models are rerun to find the new risk profile. This process is iterative.

These results show that the methodology gives useful measures for the risk of guarantees. The results show that the risk of project finance guarantees can be rigorously quantified and there are many useful intermediate results (such as cash flow profiles) that can lend insight to officials who are structuring or managing such programs.

ENDNOTES

The study was conducted by the Capital Markets Company with sponsorship from the Kingdom of Sweden, represented by Riksgäldskontoret (the Swedish National Debt Office).

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[1]For a symmetric probability density function, there is exactly a 50% chance that the loss will be greater than the expected loss. For a skewed distribution, the probability of the loss being greater than EL is a little less than 50%, but associating the idea of EL and 50% probability is a useful introduction.

[2]"A" rated banks generally reserve to a 99.9% confidence level per year.

[3]That is, unless the correlation between payments equals 1.

[4]For clarity, no details are included here on multiyear effects.

[5]GDP and CPI are scaled so that they equal 1 in the first year of the simulations.

REFERENCE

Marrison, C.I. "Design of Robust Control Laws for Uncertain Dynamic Systems Using Stochastic Robustness Metrics." Ph.D. dissertation, Princeton University, 1995.